Fundamentals of Structural Design
Part of Steel Structures

Civil Engineering for Bachelors
133FSTD

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Syllabus of lectures

1. Introduction, history of steel structures, the applications and some representative structures, production of steel
2. Steel products, material properties and testing, steel grades
3. Manufacturing of steel structures, welding, mechanical fasteners
4. Safety of structures, limit state design, codes and specifications for the design
5. Tension, compression, buckling
6. Classification of cross sections, bending, shear, serviceability limit states
7. Buckling of webs, lateral-torsional stability, torsion, combination of internal forces
8. Fatigue
9. Design of bolted and welded connections
10. Steel-concrete composite structures
11. Fire and corrosion resistance, protection of steel structures, life cycle assessment

Scope of the lecture

- Global analysis of structures
  - First or second order analysis?
  - Elastic or plastic analysis?
- Classification of cross-sections
- Elements in bending and shear
  - Ultimate limit states
  - Lateral-torsional instability
  - Serviceability limit states
  - Castellated beams
- Bi-axial bending

Global analysis of structures

Global analysis of structures is the procedure which calculates the internal forces and deformations of the structure

- First order or second order analysis?
- Elastic or plastic analysis?

Imperfections

There are three types of imperfections in the structure

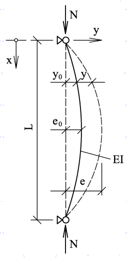
- Geometric imperfections
  bow shaped elements
- Material imperfections
  residual stress
- Structural imperfections
  random eccentricity at joints

These are introduced into calculation as equivalent geometric imperfections

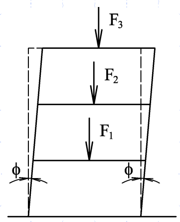
Imperfections

There are two types of imperfections:

- Element
  Are introduced as initial bow shape
  Usually not included in the analysis but accounted for by using buckling reduction factors in element check



- Global
  Are represented as sway deformation of the frame
  They are introduced as equivalent horizontal forces applied on perfect structure (without sway deformation)



Sway frame imperfections

The rotation of columns is equal to

$$\phi = \alpha_h \, \alpha_m \, \phi_0$$

where

$$\phi_0 = \frac{1}{200} \text{ rad}$$

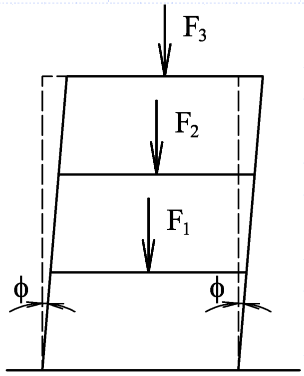
and

$$\alpha_m = \sqrt{0{,}5\left(1 + \frac{1}{m}\right)}$$

$$\alpha_h = \frac{2}{\sqrt{h}} \quad \text{but} \quad \frac{2}{3} \le \alpha_h \le 1{,}0$$

m is number of columns in a single row

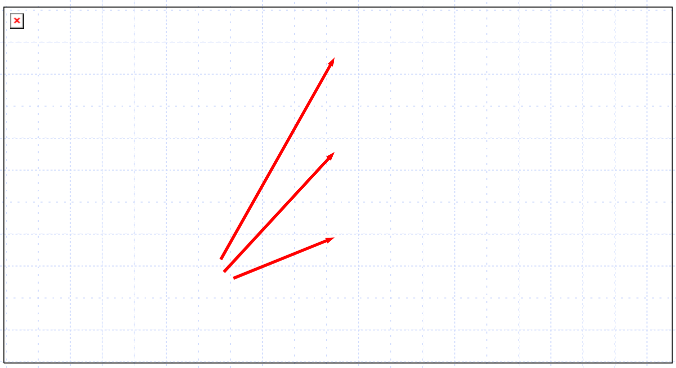
h is height of the structure



Global imperfection of a frame

Equivalent horizontal forces

The sway imperfection of the structure is replaced by equivalent horizontal forces applied to perfect structure (without sway deformation)



Global imperfection of a frame is replaced by equivalent horizontal forces

Clasification of frames

The frames are classified according to α_{cr}

$$\alpha_{cr} = \frac{V_{cr}}{V_{Ed}}$$

V_{Ed} is vertical load applied to the frame

V_{cr} is critical load of the frame

$\alpha_{cr} < 10 \Rightarrow$ sway frame $\Rightarrow$ second order theory

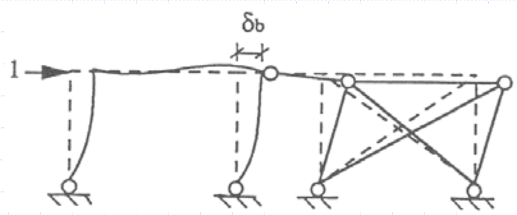
the frame is not very rigid (with respect to sway deformations), large sway deformations are expected and second order analysis (equilibrium considering the deformed shape of the structure) must be used

$\alpha_{cr} > 10 \Rightarrow$ non-sway frame $\Rightarrow$ first order theorythe frame is sufficiently rigid, therefore the sway deformation can be neglected and first order analysis (equilibrium considering the undeformed shape of the structure) can be used

Braced frames

When the frame is supported by other (rigid) structure, the sway deformation is not important and it can be considered as non-sway frame

First order theory can be used



Braced frame

Global analysis of frames

Methods of analysis

- First-order theory = geometrically linear
  - internal forces are calculated on theoretical geometry of structure
  - principle of superposition is valid
  - stability: separate buckling check of each element is performed
- Second-order theory = geometrically non-linear
  - internal forces calculated on deformed structure
  - principle of superposition is not valid (several load combinations must be solved separately)

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Scope of the lecture

Global analysis of structures

First or second order analysis?

➡ Elastic or plastic analysis?

Classification of cross-sections

Elements in bending and shear

Ultimate limit states

Lateral-torsional instability

Serviceability limit states

Castellated beams

Bi-axial bending

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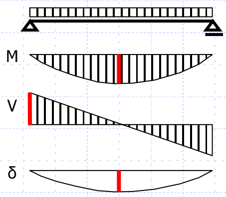
Global analysis of structures

Elastic or plastic analysis?

- Statically determined structures
  - Elastic analysis
    this is the only choice



$$M_{Ed} = \frac{q\,L^2}{8}$$

$$V_{Ed} = \frac{q\,L}{2}$$

$$\delta = \frac{5}{384}\frac{q\,L^4}{E\,I_y}$$

Example of elastic analysis

- Statically indeterminated structures
  - Elastic analysis
    load bearing capacity of the structure is predicted from the yield limit which is not exceeded at any location on the structure
    Can be used anytime for any structure
  - Plastic analysis
    Plastic hinges may develop
    Some requirements must be fulfilled:
    Sufficient ductility of the material (as large strains are observed in plastic hinges)
    The section must allow large strain before buckling occurs (most not be very slender)

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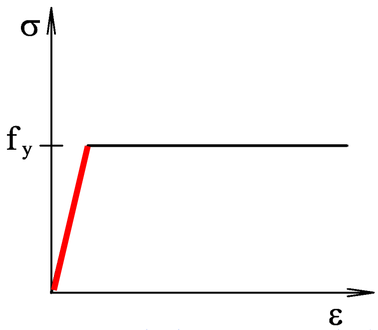
Elastic analysis

The material satisfies Hooke's law

$$\sigma = E\,\varepsilon$$

Valid for stress is smaller than the yield limit f_y



Idealised stress-strain diagram

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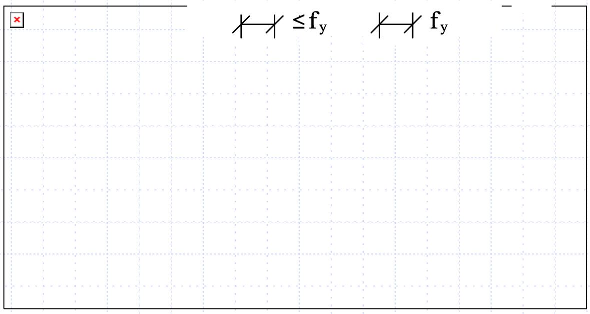
Step-by-step plastification of I cross-section

The picture shows change from elastic to fully plastic stress distribution for increasing bending moment, i.e. development of plastic hinge

Sufficient rotation capacity of cross section and ductility of steel are necessary



Cross-section: Elastic Elastic-plastic Plastic

Plastic analysis

Global plastic analysis of structure

- can be used only for indetermined (redundant) structures
- include development of plastic mechanism

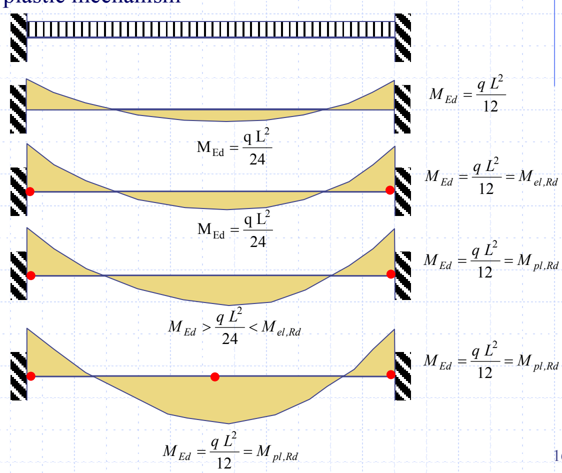
Structure:

Stage 1, elastic,
stress smaller than f_y

Stage 2, elastic,
stress equal to f_y
yielding at the supports starts

Stage 3, elastic-plastic
plastic hinges form at supports,
the middle part is still elastic

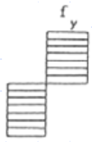
Stage 4, plastic,
plastic hinge also at mid-span,
near collapse



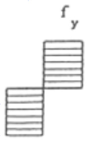
Global analysis and utilization of cross sections

- Elastic analysis - elastic stress in the critical section
  - Distribution of internal forces based on elastic analysis
  - Elastic utilization of cross section

- Elastic analysis - plastic stress in the critical section
  - Distribution of internal forces is based on elastic analysis
  - Plastic utilization of the most loaded cross section
  - Plastic redistribution of internal forces is not possible



- Plastic analysis - plastic stress in the critical section
  - Distribution of internal forces is based on plastic analysis
  - Plastic hinges create in the structure
  - Plastic redistribution of internal forces occurs
  - Sufficient rotation capacity of the hinges is necessary



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Scope of the lecture

Global analysis of structures
- First or second order analysis?
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➡ Classification of cross-sections

Elements in bending and shear
- Ultimate limit states
- Lateral-torsional instability
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Bi-axial bending

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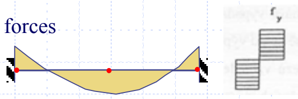
Classification of cross sections

Class 1:

Several plastic hinges

Sufficient rotation capacity for plastic redistribution of inner forces

⇒ Plastic analysis - plastic check

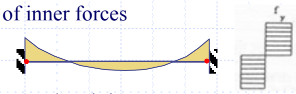


Class 2:

Plastic hinge

Limited rotation capacity of plastic hinge, no redistribution of inner forces

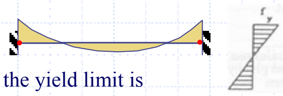
⇒ Elastic analysis - plastic check



Class 3:

It is possible to reach the yield limit in lower/upper edge fibers, when it is exceeded, buckling in the compression zone occurs

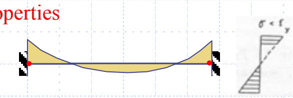
⇒ Elastic analysis - elastic check



Class 4:

Slender section, buckling in compression is observed before the yield limit is reached

⇒ Elastic analysis - elastic check with effective section properties



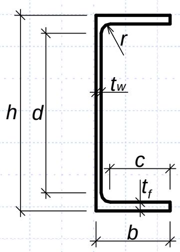
Definition of class of cross-section

Classification needs to be made for all parts with compression stress (web, flanges)

The b/t (width-to-thickness) ratio needs to be evaluated for each part and should be compared to limits for the corresponding class

- b ... width of the part (web, flange)
- t ... thickness of the part

The highest class (= $\sigma_{cr,min}$) gives the classification of the section

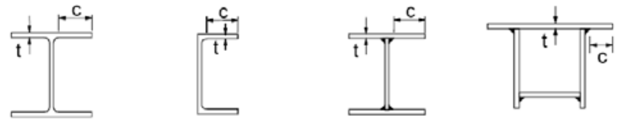


web: $\frac{d}{t_w}$

flange: $\frac{c}{t_f}$

Table for classification of outstand flanges

Outstand flanges



Class	Part subject to compression	Part subject to bending and compression: Tip in compression	Part subject to bending and compression: Tip in tension
Stress distribution in parts (compression positive)			
1	$c/t \le 9\varepsilon$	$c/t \le \frac{9\varepsilon}{\alpha}$	$c/t \le \frac{9\varepsilon}{\alpha\sqrt{\alpha}}$
2	$c/t \le 10\varepsilon$	$c/t \le \frac{10\varepsilon}{\alpha}$	$c/t \le \frac{10\varepsilon}{\alpha\sqrt{\alpha}}$
Stress distribution in parts (compression positive)			
3	$c/t \le 14\varepsilon$	$c/t \le 21\varepsilon\sqrt{k_\sigma}$ For k_σ see EN 1993-1-5	

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

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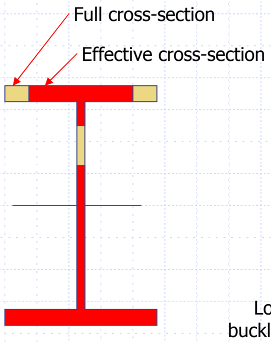
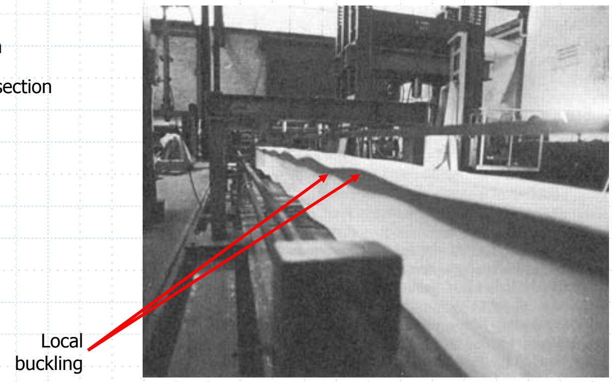
Class 4 cross-sections

Buckling of parts in compression occurs before yield limit is reached

The resistance of these parts is reduced as the stress is non-uniform there, reduced in unstiffened parts where buckling is observed

For calculation, this is replaced with assumption of uniform stress but the section is reduced - effective section is created





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Elements loaded by bending moment

- Floor and roof beams
  - usually form perpendicular grid, i.e. system of secondary beams (these directly support the concrete slab or roof cladding) and primary beams (these support the secondary beams)
- Frames
  - the elements are loaded by combination of axial force and bending moment, therefore are not typical beams, but there is a lot of similarities
- Castellated beams
  - are made from hot-rolled sections cut along the zig-zag line and welded together
  - the design procedure is quite different from "standard" beams, but these are also loaded by bending moment
- Latticed beams (trusses)
  - these are made from elements resisting tension and compression, will not be considered here
- Composite beams
  - the compression is transferred by concrete slab on top of steel beams, there is special lecture about composite structures in FSTD

Elements loaded by bending moment

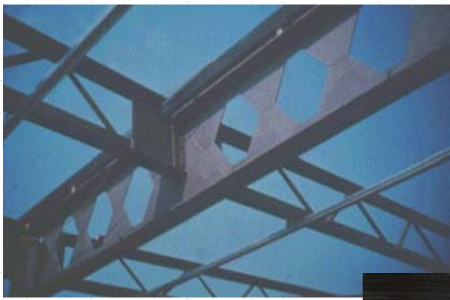
Industrial building - beams in the technological platform

Construction of a multi-storey building, system of primary and secondary beams is visible

Elements loaded by bending moment

Steel beams of a road bridge

Elements loaded by bending moment



Castellated beam - hexagonal openings

Castellated beam - Angelina beams

Castellated beam - circular openings

Elements loaded by bending moment

Beams and tapered columns of a single-storey building

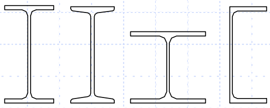
Elements loaded by bending moment

Haunched beams of single-storey building

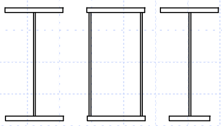
Elements loaded by bending moment

Typical cross-sections

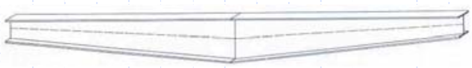
- hot-rolled



- welded



- others
  - castellated
  - non-symmetrical beams for slim floors
  - tapered, haunched, …



Tapered beam

Hot-rolled section for slim floors

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Resistance

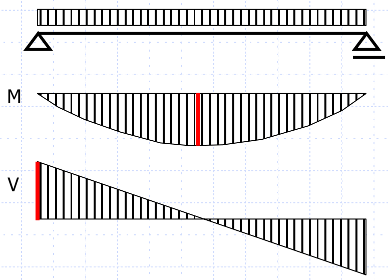
Bending moment resistance M_{Rd}

the resistance is calculated for the corresponding class of the cross-section

Class 1,2 $$M_{pl,Rd} = \frac{W_{pl}\, f_y}{\gamma_{M0}} \geq M_{Ed}$$

Class 3 $$M_{el,Rd} = \frac{W_{el}\, f_y}{\gamma_{M0}} \geq M_{Ed}$$

Class 4 $$M_{b,Rd} = \frac{W_{eff}\, f_y}{\gamma_{M0}} \geq M_{Ed}$$



W_{pl} plastic section modulus

W_{el} elastic section modulus

W_{eff} effective section modulus taking into accoun the effect of local buckling, details will not be given in FSTD

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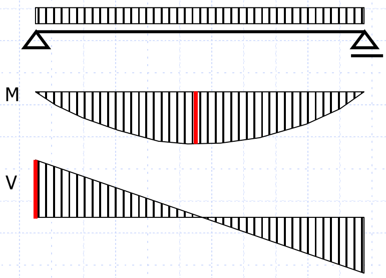
Resistance

Shear resistance V_{Rd}

plastic shear resistance is used

$$V_{pl,Rd} = \frac{A_v\, f_y}{\sqrt{3}\,\gamma_{M0}} \geq V_{Ed}$$

A_v shear area, generally the area of the beam web



In this case, the bending and shear resistance checks are independent as the maximum values of M_{Ed} and V_{Ed} are not at the same location of the beam

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Combination of shear and bending

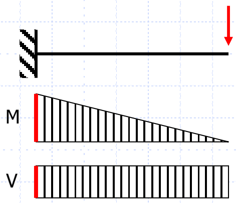
The resistance of combined shear and bending needs to be considered when shear force and bending moment appear in the same cross-section

- "Small shear force":

$V_{Ed} \leq 0{,}5\ V_{pl,Rd}$ $\Rightarrow$ combination M+V is neglected and independent check for bending and shear should be performed (as previous)

- "Large shear force":

$V_{Ed} > 0{,}5\ V_{pl,Rd}$ $\Rightarrow$ combination M+V has to be considered



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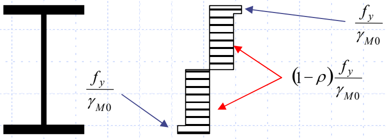
Combination of shear and bending
"Large shear force"

Reduction of strength of shear area is reduced because the presence of shear stress by (1-ρ) factor

$$\rho = \left(\frac{2\, V_{Ed}}{V_{pl,Rd}} - 1 \right)^2$$

Plastic calculation of bending moment resistance is used for all classes (but must not exceed the bending resistance of the cross section without influence of the shear force) The calculation is based on the following stress distribution



For symmetrical cross-sections (typically, hot-rolled I sections) the following formula can be used

$$M_{V,pl,Rd} = \left(W_{pl,y} - \frac{\rho\, A_V^{\,2}}{4\, t_w} \right) \frac{f_y}{\gamma_{M0}} \geq M_{Ed}$$

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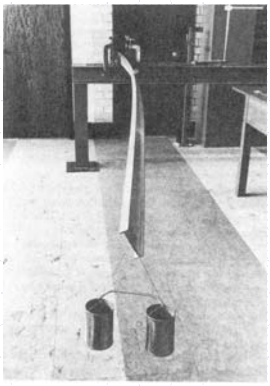
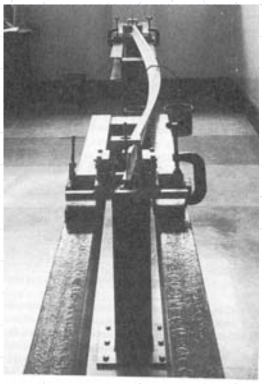
Lateral-torsional buckling

The compression flange is not restrained in lateral direction, therefore lateral movement occurs

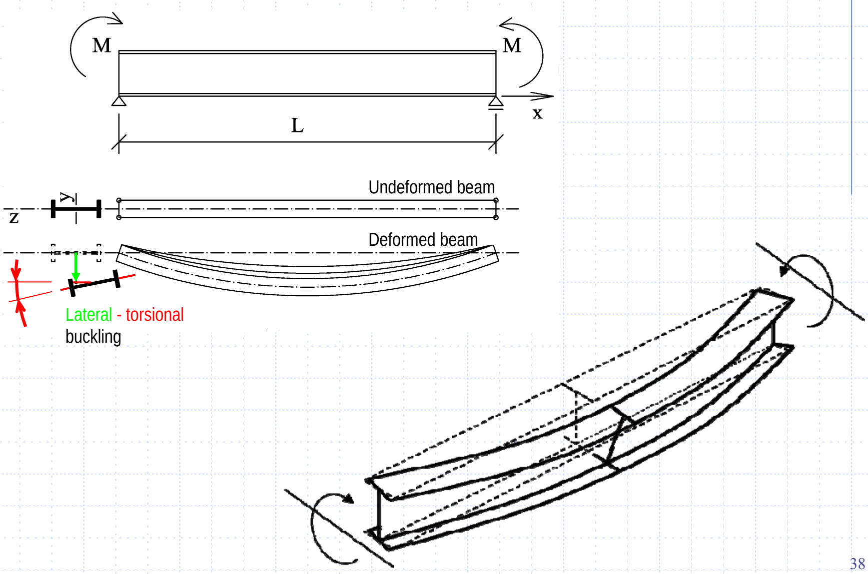


Lateral torsional buckling
of a cantilever,
the lower flange is unrestrained



Lateral torsional buckling
of simply supported beam,
the upper flange is unrestrained

Lateral-torsional buckling



Resistance to bending with lateral-torsional buckling

The resistance is given by

$$M_{b,Rd} = \frac{\chi_{LT} W_y f_y}{\gamma_{M1}} = \chi_{LT} M_{c,Rd}$$

where

$W_y = W_{pl,y}$ for Class 1, 2 cross section

$W_y = W_{el,y}$ for Class 3 cross section

$W_y = W_{eff,y}$ for Class 4 cross section

Relative slenderness is used to evaluate the buckling reduction factor χ_{LT}

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{Rk}}{M_{cr}}} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

Resistance to lateral-torsional buckling is not subject of calculation in FSTD

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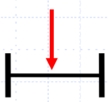
Beams not subjected to lateral-torsional buckling

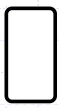
Lateral-torsional buckling can not occur in the following situations

- Lateral restrain of compression flange at short spacing L_1 or even continuous restrain

- Bending about the minor axis of the cross-section



- Hollow cross sections (high stiffness in torsion prevents lateral-torsional buckling)



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Selection of section for beams

Hot-rolled sections are preferred (approx. to 400 mm height)
- IPE or I sections are the best choice, they are optimized for beams
- In cases when the beams are not restrained in lateral direction (lateral torsional instability), wide flange sections are used (HEA)

Welded sections are slender and result in lower weight of the structure, they are designed for beams higher than approx. 600 mm

HEB sections are not good choice, they are good for columns

Choice of steel properties:
- When resistance governs, high yield limit f_y is convenient to get smaller sections and therefore lower weight ⇒ use higher steel grade (S355, S420, S460)
- When deflection governs, high yield limit has no effect as the deflection depends on modulus of elasticity E ⇒ use standard steel grades (S235, S275) which is cheaper than high strength steel

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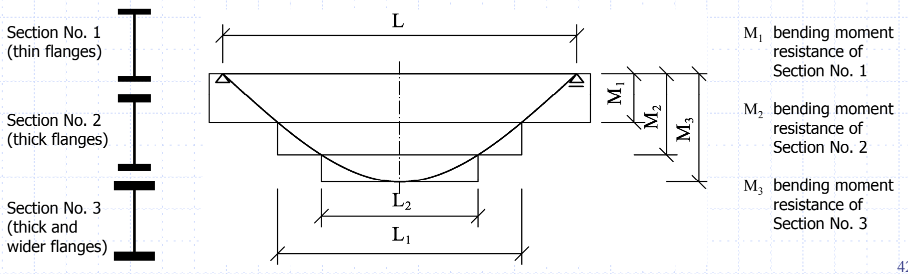
Economical design

Hot rolled I sections
- Optimized for M and V
- Slenderness of web/flanges is limited by manufacturing process
- Good choice for short spans, up to height of section approx. 400 mm

Welded I sections
- Higher production cost, should be used when height of the section exceeds approx. 600 mm
- Possible gradation of resistance according to moment distribution



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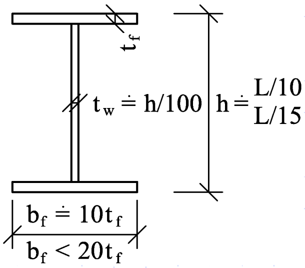
Dimensions of welded I cross-section

Criterion of efficiency is W/m (modulus / weight)

Thin web, “big” flanges

- Web stiffeners might be necessary for slender web to reduce web buckling
- Higher production cost, better efficiency, suitable for large spans
- Can be chambered to reduce deflection from the dead load



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Serviceability limit states

Deflections

- are limited for aesthetic reasons
- can be also a source of problems: cracks in brick walls, floor and wall tiles, glass façade and other brittle elements
- shear deformation can be neglected in most cases
- deflection caused by non-uniform temperature must be considered

Vibrations

- lead to discomfort of people using the building

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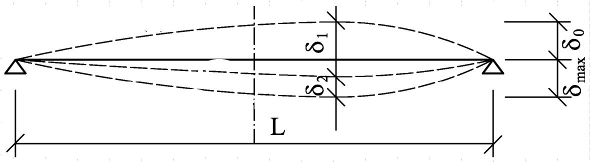
Deflections



Important section property: bending stiffness EI_y
The steel grade has no effect on the deflection of the beam

The deformation consist of several parts

δ_0 …… chamber of beam (created during the manufacturing to reduce the resultant deflection)
δ_1 …… deflection due to dead load
δ_2 …… deflection due to variable load
δ_{max} …. resultant deflection

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Recommended limits for deflections

The limits are not included in Eurocode but reference to national annex is given

In Czech Republic, the following limits are recommended:

Load	Floor	Roof
total load	L/250	L/200
variable load	L/300 (L/350)	L/250

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Vibrations

Dynamic calculation is necessary to calculate the natural frequency

It is done usually for complicated structures

For simple buildings, simplified check is always used, see below

Standard floor structures

- Natural frequency should be greater than 3 Hz
- → deflection should be smaller than 28 mm

Gymnasiums, dancing halls

- Natural frequency should be greater than 5 Hz
- → deflection should be smaller than 10 mm

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Scope of the lecture

- Global analysis of structures
  - First or second order analysis?
  - Elastic or plastic analysis?
- Classification of cross-sections
- Elements in bending and shear
  - Ultimate limit states
  - Lateral-torsional instability
  - Serviceability limit states
  - ➡ Castellated beams
- Bi-axial bending

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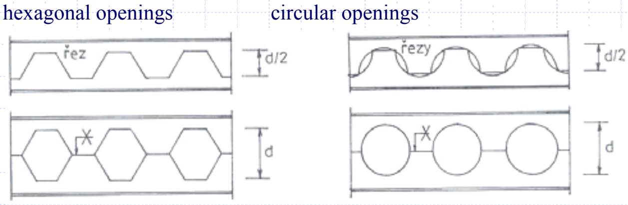
Castellated beams

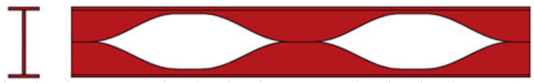
The castellated beams are manufactured from H or I beams, cut through the web along a zig-zag or sinusoidal line (two cuts are necessary to get the circular openings). The two T sections are offset and welded together providing a beam approx. 1,5 times deeper than the original profile.

Typical cross-sections

hexagonal openings circular openings



Angelina beams (produced by Arcelor Mittal)



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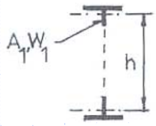
Castellated beams

Plastic design (simplification)

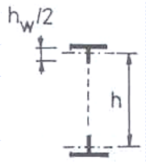
bending moment resistance

$$M_{pl,Rd} = \frac{A_1\, h\, f_y}{\gamma_{M0}}$$



shear resistance

$$V_{pl,Rd} \cong \frac{0{,}9\, h_w\, t_w\, f_y}{\sqrt{3}\, \gamma_{M0}}$$



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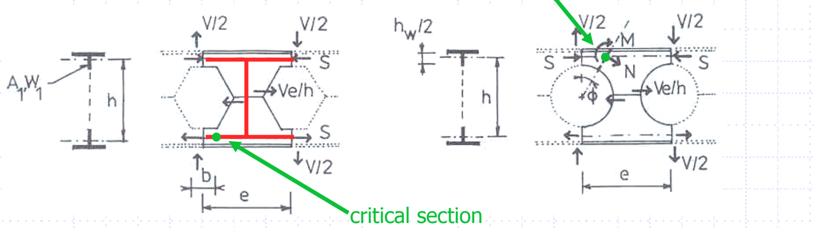
Castellated beams

Elastic design (Vierendeel beam model)

- the beam is replaced by sub-frames connected by hinges
- the internal forces acting in the fictitive connections can be simply calculated
- elastic resistance check can be performed
- details can be found in technical literature
- software exists

$$\sigma = \frac{M_{Ed}}{A_1\, h} \pm \frac{V\, b}{4\, W_1}$$

critical section needs to be found, depending on the angle ϕ



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Scope of the lecture

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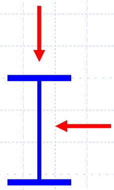
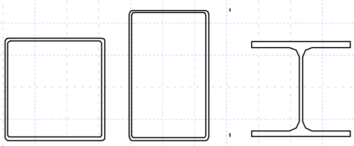
Bi-axial bending



Less frequent
- load applied in two directions
- free structures in space (masts, …)

Suitable sections:
- high I_y and I_z, W_y and W_z
- wide flange I sections, hollow sections



Bi-axial bending

$$\left(\frac{M_{y,Ed}}{M_{c,y,Rd}}\right)^{\alpha} + \left(\frac{M_{z,Ed}}{M_{c,z,Rd}}\right)^{\beta} \leq 1$$

$M_{y,Ed}$, $M_{z,Ed}$ bending moments acting about y and z axes

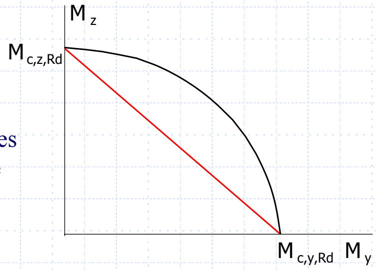
$M_{c,y,Rd}$, $M_{c,z,Rd}$ bending moment resistances

It is possible to take into account

$\alpha = \beta = 1$

(conservative approach)

Accurate method for various cross section shapes (i.e. the values of α and β) is given in Eurocode



Interaction diagram for bi-axial bending

Thank you for your attention